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Electric dipole moments of charged leptons in the split fermion scenario in the two Higgs doublet model

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Abstract. We predict the charged lepton electric dipole moments in the split fermion scenario in the framework of the two Higgs doublet model. We observe that the numerical value of the muon (tau) electric dipole moment is of the order of the magnitude of $10^{-22}\,e\,\mathrm{cm}$ ($10^{-20}\,e\,\mathrm{cm}$) and there is an enhancement in the case of two extra dimensions, especially for the tau lepton electric dipole moment.

1 Introduction

The existence of the electric dipole moments (EDMs) of fermions depends on the CP-violating interactions. The complex Cabibbo–Kobayashi–Maskawa (CKM) matrix elements cause CP-violation in the standard model (SM); however, the estimated fermion EDMs are negligibly small, if the above complex phases are considered. This stimulates one to investigate these physical quantities in the framework of the new models beyond the SM, such as multi-Higgs doublet models (MHDM), the supersymmetric model (SUSY) [1], etc.

In the literature there exist experimental results on the fermion EDMs: $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}\,e\,\mathrm{cm}$ [2], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}\,e\,\mathrm{cm}$ [3] and $d_\tau = (3.1) \times 10^{-16}\,e\,\mathrm{cm}$ [4], respectively, and on the neutron EDM $d_N < 1.1 \times 10^{-25}\,e\,\mathrm{cm}$ [5].

Extensive theoretical work has been done on the EDMs of fermions. The quark EDMs have been estimated in several models [6] and the EDMs of nuclei, deutron, neutron and some atoms have been studied extensively [7]. The lepton electric dipole moments have been predicted in various studies [8–11]. In [8] the lepton electric dipole moments have been analyzed in the framework of the seesaw model. Reference [9] was devoted to the EDMs of the leptons in the model III version of the 2HDM and d_e has been predicted to be of the order of the magnitude of $10^{-32}\,e\,\mathrm{cm}$. The work [10] was related to the lepton EDM moments in the framework of the SM with the inclusion of non-commutative geometry. Furthermore, the effects of non-universal extra dimensions on the electric dipole moments of fermions in the two Higgs doublet model have been estimated in [11].

This work is devoted to the prediction of the lepton EDMs in the two Higgs doublet model in which the flavor changing (FC) neutral current vertices at the tree level are permitted and the CP-violating interactions are carried by complex Yukawa couplings. Furthermore, we respect the split fermion scenario where the hierarchy of fermion masses is coming from the overlap of the fermion Gaussian profiles in the extra dimensions. The split fermion scenario has been studied in several works in the literature [12–19]. In [12] an alternative view of the fermion mass hierarchy has been introduced by assuming that the fermions were located at different points in the extra dimensions and this geometric interpretation resulted in exponentially small overlaps of their wavefunctions. The separation of fermions in the extra dimensions forbids the local couplings between quarks and leptons and this can ensure a solution also to the proton stability. Reference [13] was devoted to the locations of left and right handed components of fermions in the extra dimensions and their roles in the mechanism of the Yukawa hierarchies. The constraint on the split fermions in the extra dimensions has been obtained by considering leptonic W decays and the lepton violating processes in [14]. The discussion of CP-violation in the quark sector in the split fermion model was done in [15]. Reference [16] was related to the new configuration of split fermion positions in a single extra dimension and the physics of kaon, neutron and B/D mesons to find stringent bounds on the size of the compactification scale $1/R$. The contributions due to the split fermion scenario on the rare processes have been examined in [17] and the shapes and overlaps of the fermion wave functions in the split fermion model have been studied in [18].

In the present work, we consider the EDMs of charged leptons by assuming that they have Gaussian profiles in the extra dimensions. First, we study the EDMs of charged leptons in a single extra dimension, using the estimated location of them. Then, we assume that the number of extra dimensions is two and the charged leptons are restricted to the fifth extra dimension, with non-zero Gaussian profiles. As a final analysis, we predict the EDMs of charged

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leptons by taking non-zero Gaussian profiles in both extra dimensions.

In the numerical calculations, we observe that the electron EDM d_e is at the order of $10^{-32}\, e\,\mathrm{cm}$, and this is too small to study the additional effects due to the extra dimensions. The numerical value of d_μ (d_τ) is at most of the order of the magnitude of 10^{-22} (10^{-20}) $e\,\mathrm{cm}$ in the case that the leptons have non-zero Gaussian profiles in the first extra dimension, for two extra dimensions, for large values of the compactification scale $1/R$ and for intermediate values of Yukawa couplings.

This paper is organized as follows: In Sect. 2, we present EDMs of charged leptons in the split fermion scenario, in the two Higgs doublet model. Section 3 is devoted to a discussion and our conclusions.

2 Electric dipole moments of charged leptons in the split fermion scenario in the two Higgs doublet model

The fermion EDM is carried by the CP-violating fermion–fermion–photon effective interaction and, for quarks (for charged leptons), the complex CKM matrix (possible lepton mixing matrix) elements is the possible source of this violation, in the framework of the SM. The estimated tiny numerical values of EDMs of fermions in the SM stimulates one to go beyond, and the model III version of the 2HDM is one of the candidates to get relatively large EDM values, since the FC neutral currents (FCNC) are permitted at tree level and the new Yukawa couplings can be complex in general. Furthermore, the addition of the spatial extra dimensions brings additional contributions sensitive to the compactification scale $1/R$ where R is the radius of the compactification. Here we take the effects of extra dimensions into account and we follow the idea that the hierarchy of lepton masses is coming from the lepton Gaussian profiles in the extra dimensions.

The Yukawa Lagrangian responsible for the lepton EDM in a single extra dimension, respecting the split fermion scenario, reads

$$\mathcal{L}_{\mathrm{Y}} = \xi^{E}_{5\,ij} \bar{\hat{l}}_{iL} \phi_2 \hat{E}_{jR} + \mathrm{h.c.}, \tag{1}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, and ϕ_2 is the new scalar doublet. Here $\hat{l}_{iL}$ ($\hat{E}_{jR}$), with family indices i, j, are the zero mode[1] lepton doublets (singlets) with Gaussian profiles in the extra dimension y and they read

$$\begin{aligned}\hat{l}_{iL} &= N\, \mathrm{e}^{-(y-y_{iL})^2/2\sigma^2}\, l_{iL}, \\ \hat{E}_{jR} &= N\, \mathrm{e}^{-(y-y_{jR})^2/2\sigma^2}\, E_{jR},\end{aligned} \tag{2}$$

with the normalization factor $N = \frac{1}{\pi^{1/4}\sigma^{1/2}}$. l_{iL} (E_{jR}) are the lepton doublets (singlets) in four dimensions. The parameter σ is the Gaussian width of the leptons with the property $\sigma \ll R$, $y_{i(L,R)}$ are the fixed position of ith left (right) handed lepton in the fifth dimension. The positions of left and right handed leptons are obtained by taking the observed masses into account [13]. The idea is that the lepton mass hierarchy is due to the relative positions of the Gaussian peaks of the wave functions located in the extra dimension [12,13]. By assuming that the lepton mass matrix is diagonal, one possible set of locations for the lepton fields reads (see [13] for details)

$$P_{l_i} = \sqrt{2}\,\sigma \begin{pmatrix} 11.075 \\ 1.0 \\ 0.0 \end{pmatrix}, \quad P_{e_i} = \sqrt{2}\,\sigma \begin{pmatrix} 5.9475 \\ 4.9475 \\ -3.1498 \end{pmatrix}. \tag{3}$$

Here we choose the Higgs doublets ϕ_1 and ϕ_2 to be

$$\begin{aligned}\phi_1 &= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ \mathrm{i}\chi^0 \end{pmatrix} \right] \\ \phi_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + \mathrm{i} H_2 \end{pmatrix},\end{aligned} \tag{4}$$

with the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}; \quad \langle \phi_2 \rangle = 0, \tag{5}$$

and we collect SM (new) particles in the first (second) doublet. Notice that H_1 and H_2 are the mass eigenstates h^0 and A^0, respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 at tree level, in our case.

The new Higgs field ϕ_2, playing the main role in the existence of the charged lepton EDM, is accessible to the extra dimension and after the compactification on the orbifold S^1/Z_2, it is expanded as

$$\begin{aligned}&\phi_2(x, y) \\ &= \frac{1}{\sqrt{2\pi R}} \left\{ \phi_2^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \phi_2^{(n)}(x) \cos(ny/R) \right\},\end{aligned} \tag{6}$$

where $\phi_2^{(0)}(x)$ is the four dimensional Higgs doublet which contains the charged Higgs boson H^+, the neutral CP-even (-odd) Higgs bosons h^0 (A^0), and $\phi_2^{(n)}(x)$ are the KK modes of ϕ_2. The non-zero KK mode of the Higgs doublet ϕ_2 includes a charged Higgs of mass $\sqrt{m_{H^\pm}^2 + m_n^2}$, a neutral CP-even Higgs of mass $\sqrt{m_{h^0}^2 + m_n^2}$, and a neutral CP-odd Higgs of mass $\sqrt{m_{A^0}^2 + m_n^2}$ where $m_n = n/R$ is the mass of the nth level KK particle. Notice that the gauge field KK modes do not bring new contributions to the EDMs of charged leptons since they do not exist in the one loop diagrams (see Fig. 1).

Now, we present the vertices existing in the diagrams, after the integration over the fifth dimension. The integration of the combination $\bar{\hat{f}}_{iL\,(R)}\, S^{(n)}(x)\, \cos(ny/R)\, \hat{f}_{jR\,(L)}$,

[1] In our calculations, we take only zero mode lepton fields. See Sect. 3 for further explanation.

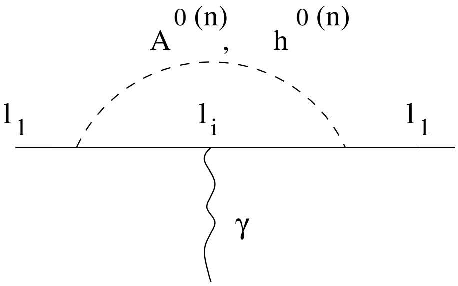

Fig. 1. One loop diagrams contribute to the EDMs of charged leptons due to neutral Higgs bosons h^0, A^0 in the 2HDM, including KK modes in a single extra dimension. Wavy lines represent the electromagnetic field and dashed lines the Higgs field where $l_{1\,(i)} = e, \mu, \tau$

$(S = h^0, A^0)$, appearing in the part of the Lagrangian (see (1)), over the fifth dimension reads

$$\int_{-\pi R}^{\pi R} \mathrm{d}y\ \bar{\hat{f}}_{iL\,(R)}\, S^{(n)}(x)\, \cos(ny/R)\, \hat{f}_{jR\,(L)} = V^{n}_{LR\,(RL)\,ij}\, \bar{f}_{iL\,(R)}\, S^{(n)}(x)\ f_{jR\,(L)}, \tag{7}$$

where the factor $V^{n}_{LR\,(RL)\,ij}$ is

$$V^{n}_{LR\,(RL)\,ij} = \mathrm{e}^{-n^2\,\sigma^2/4\,R^2}\ \mathrm{e}^{-(y_{iL\,(R)}-y_{jR\,(L)})^2/4\sigma^2} \times \cos\left[\frac{n\,(y_{iL\,(R)}+y_{jR\,(L)})}{2\,R}\right]. \tag{8}$$

Here the fields f_{iL}, f_{jR} are the four dimensional lepton fields. Therefore we can define the Yukawa couplings in four dimensions as

$$\xi^{E}_{ij}\ \left((\xi^{E\dagger}_{ij})^\dagger\right) = V^{0}_{LR\,(RL)\,ij}\,\xi^{E}_{5\,ij}\ \left((\xi^{E}_{5\,ij})^\dagger\right)/\sqrt{2\pi R}, \tag{9}$$

where $\xi^{E}_{5\,ij}$ are Yukawa couplings in five dimensions (see (1))[2].

In the case of two extra dimensions, after the compactification on the orbifold $(S^1\times S^1)/Z_2$, the new Higgs field ϕ_2 can be expanded as

$$\phi_2(x,y,z) = \frac{1}{2\pi R}\left\{\phi_2^{(0,0)}(x) + 2\sum_{n,s}^{\infty}\phi_2^{(n,s)}(x)\cos(ny/R+sz/R)\right\}, \tag{10}$$

where $\phi_2^{(n,s)}(x)$ are the KK modes of ϕ_2. Notice that the mass of KK modes of the charged (neutral CP-even, neutral CP-odd) Higgs is

$$\sqrt{m^2_{H^\pm}+m^2_n+m^2_s},\quad \left(\sqrt{m^2_{h^0}+m^2_n+m^2_s},\ \sqrt{m^2_{A^0}+m^2_n+m^2_s}\right),$$

where $m_n = n/R$ ($m_s = s/R$) is the mass of the $n(s)$th level KK particle. In the case that the leptons are restricted to only the fifth dimension, the vertex factor $V^{n}_{LR\,(RL)\,ij}$ is the same as the one in (8). On the other hand, if we assume that the leptons are accessible to both dimensions with Gaussian profiles as

$$\hat{l}_{iL} = N\,\mathrm{e}^{-\left((y-y_{iL})^2+(z-z_{iL})^2\right)/2\sigma^2}\ l_{iL},\qquad \hat{E}_{jR} = N\,\mathrm{e}^{-\left((y-y_{jR})^2+(z-z_{jR})^2\right)/2\sigma^2}\ E_{jR}, \tag{11}$$

with the normalization factor $N = \frac{1}{\pi^{1/2}\,\sigma}$, the integration of the part of the Lagrangian $\bar{\hat{f}}_{iL\,(R)}\,S^{(n,s)}(x)\,\cos(ny/R+sz/R)\,\hat{f}_{jR\,(L)}$ over the fifth and sixth extra dimensions results in the vertex factor

$$V^{(n,s)}_{LR\,(RL)\,ij} = \mathrm{e}^{-(n^2+s^2)\,\sigma^2/4\,R^2}\ \mathrm{e}^{-\left((y_{iL\,(R)}-y_{jR\,(L)})^2+(z_{iL\,(R)}-z_{jR\,(L)})^2\right)/4\sigma^2} \times\cos\left[\frac{n\,(y_{iL\,(R)}+y_{jR\,(L)})+s\,(z_{iL\,(R)}+z_{jR\,(L)})}{2\,R}\right]. \tag{12}$$

Similar to a single extra dimension case, we define the Yukawa couplings in four dimension as

$$\xi^{E}_{ij}\ \left((\xi^{E}_{ij})^\dagger\right) = V^{(0,0)}_{LR\,(RL)\,ij}\,\xi^{E}_{6\,ij}\ \left((\xi^{E}_{6\,ij})^\dagger\right)/2\pi R, \tag{13}$$

where $V^{(0,0)}_{LR\,(RL)\,ij} = \mathrm{e}^{-\left((y_{iL\,(R)}-y_{jR\,(L)})^2+(z_{iL\,(R)}-z_{jR\,(L)})^2\right)/4\sigma^2}$. Here, we present the possible positions of left and right handed leptons in the two extra dimensions by respecting the observed masses[3]. Similar to the previous discussion, we assume that the lepton mass matrix is diagonal and one of the possible set of locations for the Gaussian peaks of the lepton fields in the two extra dimensions reads

$$P_{l_i} = \sqrt{2}\,\sigma\begin{pmatrix}(8.417, 8.417)\\(1.0, 1.0)\\(0.0, 0.0)\end{pmatrix},\qquad P_{e_i} = \sqrt{2}\,\sigma\begin{pmatrix}(4.7913, 4.7913)\\(3.7913, 3.7913)\\(-2.2272, -2.2272)\end{pmatrix}, \tag{14}$$

where the numbers in the parentheses denote the y and z coordinates of the location of the Gaussian peak in the extra dimensions. Here we choose the same numbers for the y and z locations of the Gaussian peaks.

Now, we would like to present the EDMs of charged leptons with the addition of a single extra dimension where the localized leptons have Gaussian profiles. The effective EDM interaction for a charged lepton f is given by

$$\mathcal{L}_{\mathrm{EDM}} = \mathrm{i}d_f\,\bar{f}\,\gamma_5\,\sigma^{\mu\nu}\,f\,F_{\mu\nu}, \tag{15}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, "d_f" is the EDM of the charged lepton and it is a real number by hermiticity. With the assumption that there is no CKM type

[2] In the following we use the dimensionful coupling $\bar{\xi}^{E}_{\mathrm{N}}$ with the definition $\xi^{E}_{\mathrm{N},ij} = \sqrt{\frac{4\,G_{\mathrm{F}}}{\sqrt{2}}}\,\bar{\xi}^{E}_{\mathrm{N},ij}$, where N denotes the word "neutral".

[3] The calculation is similar to the one presented in [13] which is done for a single extra dimension.

lepton mixing matrix and ignoring possible CP-violating LFV interactions due to the lepton–lepton KK mode–Higgs KK mode vertices, only the new neutral Higgs part gives a contribution to their EDMs and f-lepton EDM "d_f" ($f = e, \mu, \tau$) can be calculated as a sum of contributions coming from neutral Higgs bosons h_0 and A_0,

$$
\begin{aligned}
d_f = & -\frac{\mathrm{i}G_\mathrm{F}}{\sqrt{2}}\frac{e}{32\pi^2}\frac{Q_\tau}{m_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},l\tau})^2 \\
& - (\bar{\xi}^{D}_{\mathrm{N},\tau l})^2)\,((F_1(y_{h_0}) - F_1(y_{A_0})) \\
& + 2\sum_{n=1}^{\infty} \mathrm{e}^{-n^2\sigma^2/2R^2} \\
& \quad \times c_n(f,\tau)\,c'_n(f,\tau)\,\left(F_1(y^n_{h_0}) - F_1(y^n_{A_0})\right))\,,
\end{aligned} \tag{16}
$$

for $f = e, \mu$ and

$$
\begin{aligned}
d_\tau = & -\frac{\mathrm{i}G_\mathrm{F}}{\sqrt{2}}\frac{e}{32\pi^2} \\
& \times \left\{\frac{Q_\tau}{m_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},\tau\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau\tau})^2)\,((F_2(r_{h_0}) - F_2(r_{A_0}))\right. \\
& \left. + 2\sum_{n=1}^{\infty}\mathrm{e}^{-n^2\sigma^2/2R^2}c^2_n(\tau,\tau)\left(F_2(r^n_{h_0}) - F_2(r^n_{A_0})\right)\right) \\
& - Q_\mu\frac{m_\mu}{m^2_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},\mu\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau\mu})^2) \\
& \quad \times ((r_{h_0}\ln(z_{h_0}) - r_{A_0}\ln(z_{A_0})) \\
& + 2\sum_{n=1}^{\infty}\mathrm{e}^{-n^2\sigma^2/2R^2} \\
& \quad \times c_n(\mu,\tau)c'_n(\mu,\tau)\left(r^n_{h_0}\ln(z^n_{h_0}) - r^n_{A_0}\ln(z^n_{A_0})\right))\}\,,
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
c_n\,(f,\tau) &= \cos\left[\frac{n\,(y_{fR} + y_{\tau L})}{2\,R}\right], \\
c'_n\,(f,\tau) &= \cos\left[\frac{n\,(y_{fL} + y_{\tau R})}{2\,R}\right],
\end{aligned} \tag{18}
$$

for $f = e, \mu, \tau$ and the functions $F_1(w)$, $F_2(w)$ read

$$
\begin{aligned}
F_1(w) &= \frac{w\,(3 - 4\,w + w^2 + 2\ln w)}{(-1+w)^3}, \\
F_2(w) &= w\ln w + \frac{2\,(-2+w)\,w\ln\frac{1}{2}(\sqrt{w} - \sqrt{w-4})}{\sqrt{w\,(w-4)}},
\end{aligned} \tag{19}
$$

with $y^n_S = \frac{m^2_\tau}{m^2_S + n^2/R^2}$, $r^n_S = \frac{1}{y^n_S}$ and $z^n_S = \frac{m^2_\mu}{m^2_S + n^2/R^2}$, $y_S = y^0_S$, $r_S = r^0_S$ and $z_S = z^0_S$; Q_τ, Q_μ are the charges of τ and μ leptons respectively. In (16) we take into account only the internal τ-lepton contribution respecting our assumption that the Yukawa couplings $\bar{\xi}^E_{\mathrm{N},ij}$, $i,j = e,\mu$, are small compared to $\bar{\xi}^E_{\mathrm{N},\tau\,i}$ $i = e,\mu,\tau$ due to the possible proportionality of the Yukawa couplings to the masses of leptons in the vertices. In (17) we present also the internal μ-lepton contribution, which can be neglected numerically. Notice that we make our calculations for an arbitrary photon four momentum square q^2 and take $q^2 = 0$ at the end.

Now, we present the EDMs of charged leptons with the addition of two extra dimensions for the case that the leptons are accessible to both extra dimensions:

$$
\begin{aligned}
d_f = & -\frac{\mathrm{i}G_\mathrm{F}}{\sqrt{2}}\frac{e}{32\pi^2} \\
& \times\frac{Q_\tau}{m_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},l\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau l})^2)\,((F_1(y_{h_0}) - F_1(y_{A_0})) \\
& + 4\sum_{n,s}^{\infty}\mathrm{e}^{-(n^2+s^2)\,\sigma^2/2\,R^2} \\
& \quad \times c_{2\,(n,s)}(f,\tau)\,c'_{2\,(n,s)}(f,\tau) \\
& \quad\quad \times\left(F_1(y^{(n,s)}_{h_0}) - F_1(y^{(n,s)}_{A_0})\right))\,,
\end{aligned} \tag{20}
$$

for $f = e, \mu$ and

$$
\begin{aligned}
d_\tau = & -\frac{\mathrm{i}G_\mathrm{F}}{\sqrt{2}}\frac{e}{32\pi^2} \\
& \times\left\{\frac{Q_\tau}{m_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},\tau\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau\tau})^2)\,((F_2(r_{h_0}) - F_2(r_{A_0}))\right. \\
& + 4\sum_{n,s}^{\infty}\mathrm{e}^{-(n^2+s^2)\sigma^2/2R^2} \\
& \quad \times(c_{2\,(n,s)})^2(\tau,\tau) \\
& \quad\quad \times\left(F_2(r^(n,s)_{h_0}) - F_2(r^(n,s)_{A_0})\right)) \\
& - Q_\mu\frac{m_\mu}{m^2_\tau}\,((\bar{\xi}^{D\,*}_{\mathrm{N},\mu\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau\mu})^2) \\
& \quad \times((r_{h_0}\ln(z_{h_0}) - r_{A_0}\ln(z_{A_0})) \\
& + 4\sum_{n,s}^{\infty}\mathrm{e}^{-(n^2+s^2)\sigma^2/2R^2} \\
& \quad \times c_{2\,(n,s)}(\mu,\tau)c'_{2\,(n,s)}(\mu,\tau) \\
& \quad\quad \times\left(r^{(n,s)}_{h_0}\ln(z^{(n,s)}_{h_0}) - r^{(n,s)}_{A_0}\ln(z^{(n,s)}_{A_0})\right))\Big\}\,,
\end{aligned} \tag{21}
$$

where

$$
\begin{aligned}
c_{2\,(n,s)}(f,\tau) &= \cos\left[\frac{n(y_{fR} + y_{\tau L}) + s(z_{fR} + z_{\tau L})}{2R}\right], \\
c'_{2\,(n,s)}(f,\tau) &= \cos\left[\frac{n(y_{fL} + y_{\tau R}) + s(z_{fL} + z_{\tau R})}{2R}\right],
\end{aligned} \tag{22}
$$

for $f = e, \mu, \tau$. In (20) and (21), the parameters $y^{(n,s)}_S$, $r^{(n,s)}_S$ and $z^{(n,s)}_S$ are defined by $y^{(n,s)}_S = \frac{m^2_\tau}{m^2_S + n^2/R^2 + s^2/R^2}$, $r^{(n,s)}_S = \frac{1}{y^{(n,s)}_S}$ and $z^{(n,s)}_S = \frac{m^2_\mu}{m^2_S + n^2/R^2 + s^2/R^2}$. In (20) and (21) the summation would be done over $n, s = 0, 1, 2, ...$ except $n = s = 0$.

Finally, in our calculations, we choose the Yukawa couplings complex and we used the parametrization

$$
\bar{\xi}^E_{\mathrm{N},\tau f} = |\bar{\xi}^E_{\mathrm{N},\tau f}|\mathrm{e}^{\mathrm{i}\theta_f}\,. \tag{23}
$$

Therefore, the Yukawa factors in (16), (17), (20) and (21) can be written as

$$
\left((\bar{\xi}^{D\,*}_{\mathrm{N},f\tau})^2 - (\bar{\xi}^{D}_{\mathrm{N},\tau f})^2\right) = -2\mathrm{i}\sin 2\theta_f|\bar{\xi}^{D}_{\mathrm{N},\tau f}|^2, \tag{24}
$$

where $f = e, \mu, \tau$. Here θ_f is the CP-violating parameter which is the source of the lepton EDM.

3 Discussion

This work is devoted to the analysis of the effects of extra dimensions on the EDMs of fermions in the case that the hierarchy of lepton masses is due to the lepton Gaussian profiles in the extra dimensions. The CP-violating nature of the EDM interactions needs CP-violating phases. Here, for the complex phases, we consider the complex Yukawa couplings appearing in the FCNC at tree level in the framework of the 2HDM. Notice that we do not take the internal lepton KK modes into account (see for example [13] for the calculation of the KK modes of leptons). The lepton–lepton KK mode-Higgs zero mode and lepton–lepton KK mode–Higgs KK mode vertices carry the possible LFV and CP-violating interactions. We ignored these contributions because of the difficulty arising during the summations. We expect that, for large values of the compactification scale, their effects on the physical parameters are suppressed.

The four dimensional leptonic complex couplings $\bar{\xi}^{E}_{\mathrm{N},ij}$, $i,j = e,\mu,\tau$ are the free parameters of 2HDM. We consider the Yukawa couplings $\bar{\xi}^{E}_{\mathrm{N},ij}$, $i,j = e,\mu$, as smaller than $\bar{\xi}^{E}_{\mathrm{N},\tau\, i}$, $i = e,\mu,\tau$, and we assume that $\bar{\xi}^{E}_{\mathrm{N},ij}$ is symmetric with respect to the indices i and j. In the case that no extra dimension exists, the upper limit of $\bar{\xi}^{D}_{\mathrm{N},\tau\mu}$ is predicted to be 30 GeV (see [20] and references therein) by using the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment [21] and assuming that the new physics effects cannot exceed this uncertainty. Using this upper limit and the experimental upper bound of the Br of $\mu \to e\gamma$ decay, $\mathrm{Br} \leq 1.2 \times 10^{-11}$, the coupling $\bar{\xi}^{D}_{\mathrm{N},\tau e}$ can be restricted in the range 10^{-3}–10^{-2} GeV [9]. In our calculations we choose the numerical values of the couplings $\bar{\xi}^{E}_{\mathrm{N},\tau\mu}$ ($\bar{\xi}^{E}_{\mathrm{N},\tau e}$) around 30 GeV ($10^{-2}$ GeV). For the coupling $\bar{\xi}^{E}_{\mathrm{N},\tau\tau}$, we use the numerical values which are greater than $\bar{\xi}^{E}_{\mathrm{N},\tau\mu}$, since we have no explicit restriction region.

Here, we respect the split fermion scenario where the hierarchy of lepton masses is due to the lepton Gaussian profiles in the extra dimensions. The SM scalar H^0 is a constant profile in the extra dimensions and the mass term, which is modulated by the mutual overlap of the lepton wavefunctions, is obtained by integrating the operator $H^0 \bar{\hat{f}} \bar{f}$ over extra dimensions. This idea is the main point to fix the position of left (right) handed lepton in the extra dimensions (see [13] for details). Since the leptons are located in the extra dimensions with Gaussian profiles, the parameter $\rho = \sigma/R$, where σ is the Gaussian width of the fermions, is the free parameter of the model. The locations of lepton fields in the extra dimensions are obtained in terms of the Gaussian width σ.

In the present work we take split leptons in a single and two extra dimensions and use a possible set of locations to calculate the strength of the lepton–lepton–new Higgs scalars vertices, which play the main role in the calculation of the charged lepton EDM. First, we take a single extra dimension and use the estimated location of the leptons (see (3)) to calculate the corresponding vertices (see (7)). After that, we assume that the number of extra dimensions is two and take the leptons to be restricted to the fifth extra dimension, with non-zero Gaussian profiles. In this case the abundance of new scalar KK modes causes the increase of the EDM of charged leptons, especially the τ lepton EDM. However, the exponential suppression factor (see (8)) appearing in the summation of the KK modes causes the sum not to have a large contribution. Finally, we assume that the leptons have non-zero Gaussian profiles also in the sixth dimension and using a possible set of locations in the fifth and sixth extra dimensions (see (14)), we calculated the EDM of the charged leptons. In this case the additional exponential factor appearing in the second summation further suppresses the lepton EDM especially for the muon case.

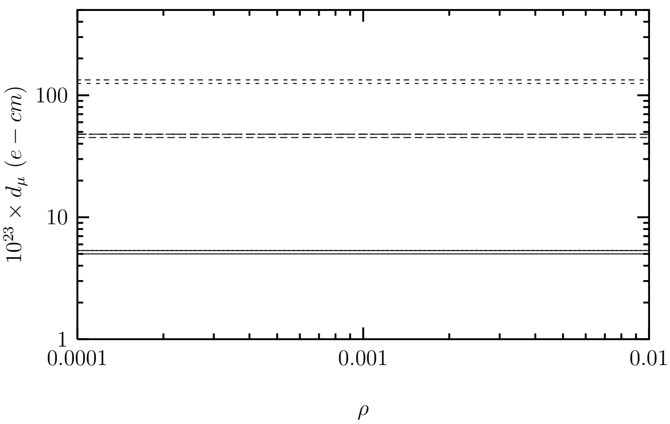

Fig. 2. d_μ with respect to the parameter ρ for $1/R = 500$ GeV, $m_{h^0} = 100$ GeV, $m_{A^0} = 200$ GeV and the intermediate value of $\sin\theta_\mu = 0.5$. The lower–upper solid (dashed, small dashed) line represents the d_μ for a single–two extra dimensions, for $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10\,(30,\ 50)$ GeV

Now, we start to estimate the charged lepton EDMs and to study the parameter ρ and the compactification scale $1/R$ dependences of these measurable quantities.

In Fig. 2, we plot EDM d_μ with respect to the parameter ρ for $1/R = 500$ GeV, $m_{h^0} = 100$ GeV, $m_{A^0} = 200$ GeV and the intermediate value of $\sin\theta_\mu = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, for $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10\,(30,\ 50)$ GeV. The EDM is slightly larger for the case that the leptons have non-zero Gaussian profiles in the first extra dimension, compared to the one where the leptons have non-zero Gaussian profiles in both extra dimensions. It is observed that d_μ is weakly sensitive to the parameter ρ in the given interval, for the chosen value of the compactification scale $1/R$. In the two extra dimensions the numerical value of d_μ is larger compared to the one single extra dimension since there is a crowd of KK modes. However, the suppression exponential factor appearing in the summations causes the contributions not to increase extremely. The numerical value of

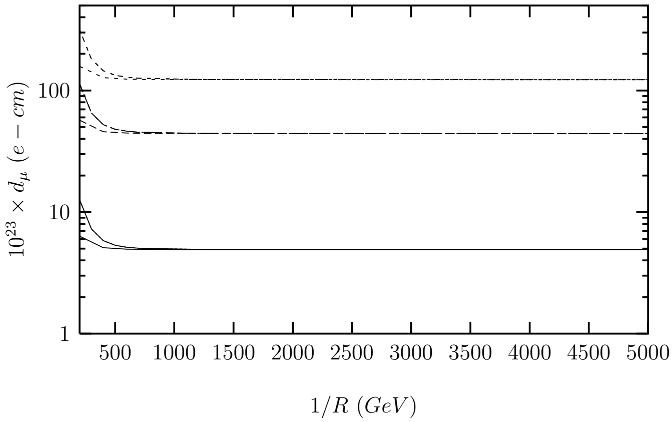

Fig. 3. d_μ with respect to the scale $1/R$, for $\rho = 0.01$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\mu = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the d_μ for a single–two extra dimensions, for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 10\,(30,\,50)\,\mathrm{GeV}$

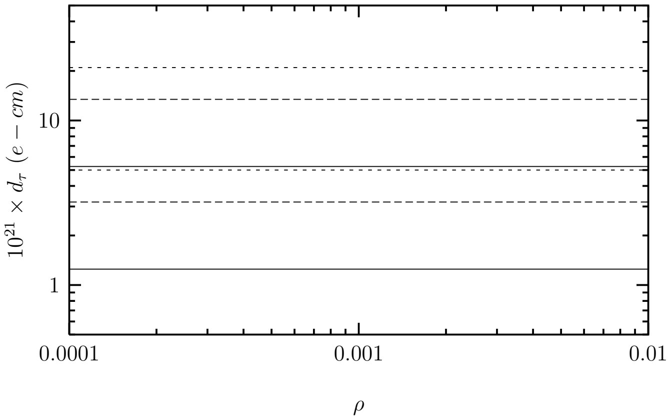

Fig. 4. d_τ with respect to the parameter ρ for $1/R = 500\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\tau = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, where the leptons have non-zero Gaussian profiles in the first extra dimension, for $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 50\,(80,\,100)\,\mathrm{GeV}$

d_μ is of the order of the magnitude of $5.0 \times 10^{-22}\,e\,\mathrm{cm}$ for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 30\,\mathrm{GeV}$, in the case that the leptons have non-zero Gaussian profiles in the first extra dimension.

Figure 3 is devoted to the EDM d_μ with respect to the compactification scale $1/R$, for $\rho = 0.01$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\mu = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, where the leptons have non-zero Gaussian profiles in the first extra dimension, for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 10\,(30,\,50)\,\mathrm{GeV}$. This figure shows that d_μ is weakly sensitive to the compactification scale $1/R$, especially for $1/R > 500\,\mathrm{GeV}$.

In Fig. 4, we present the EDM d_τ with respect to the parameter ρ for $1/R = 500\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\tau = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, where the leptons have non-zero Gaussian profiles in the first extra dimension, for $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 50\,(80,\,100)\,\mathrm{GeV}$. For the case where the leptons have non-zero Gaussian profiles in both extra dimensions, the numerical value of d_τ is almost the same as the one where the leptons have non-zero Gaussian profiles in only one extra dimension, for the two extra dimensions scenario. It is shown that d_τ is weakly sensitive to the parameter ρ in the given interval. Due to the crowd of KK modes, in the two extra dimensions, the numerical value of d_τ is almost five times larger compared to the one in the single extra dimension. The numerical value of d_τ is of the order of the magnitude of $10^{-20}\,e\,\mathrm{cm}$ for $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 80\,\mathrm{GeV}$, in the two extra dimensions.

Figure 5 represents the compactification scale $1/R$ dependence of the EDM d_τ, for $\rho = 0.01$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\tau = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, where the leptons have non-zero Gaussian profiles in the first extra dimension, for $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 50\,(80,\,100)\,\mathrm{GeV}$. For the case where the leptons have non-zero Gaussian profiles in both extra dimensions the numerical values of d_τ is almost the same as the one where the leptons have non-zero Gaussian profiles in only one extra dimension. Similar to the μ EDM case d_τ is weakly sensitive to the compactification scale $1/R$, especially for $1/R > 500\,\mathrm{GeV}$.

Now we would like to summarize our results.

(1) d_μ is weakly sensitive to the parameter ρ for $\rho < 0.01$ and the compactification scale $1/R > 500\,\mathrm{GeV}$. Due to the abundance of KK modes, in the two extra dimensions the numerical value of d_μ is slightly larger compared to the one in the single extra dimension. The numerical value of d_μ is at most of the order of the magnitude of $5.0 \times 10^{-22}\,e\,\mathrm{cm}$ for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 30\,\mathrm{GeV}$, in the case that the leptons have non-zero Gaussian profiles in the first extra dimension.

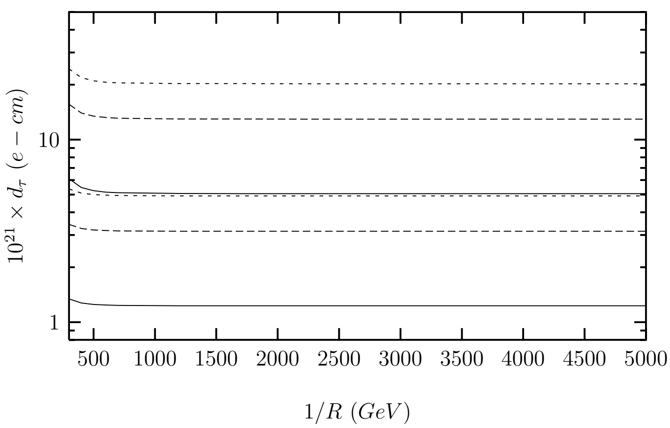

Fig. 5. d_τ with respect to the scale $1/R$, for $\rho = 0.01$, $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$ and the intermediate value of $\sin\theta_\tau = 0.5$. Here the lower–upper solid (dashed, small dashed) line represents the EDM for a single–two extra dimensions, where the leptons have non-zero Gaussian profiles in the first extra dimension, for $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 50\,(80,\,100)\,\mathrm{GeV}$

(2) d_τ is weakly sensitive to the parameter ρ for $\rho < 0.01$ and the compactification scale $1/R > 500\,\mathrm{GeV}$. The crowd of KK modes in the two extra dimensions bring additional contributions which enhance d_τ almost five times compared to the one in a single extra dimension. The numerical value of d_τ is of the order of the magnitude of $10^{-20}\,e\,\mathrm{cm}$ for $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 80\,\mathrm{GeV}$, in the two extra dimensions.
(3) The addition of the effects of the internal lepton KK modes brings an extra dependence of the physical parameters to the scale $1/R$; however, for the large values of the parameter $1/R$, hopefully, these contributions do not affect the scale $1/R$ dependence of the physical parameters.

With the help of the forthcoming most accurate experimental measurements, valuable information can be obtained about the existence of extra dimensions and the possibilities of Gaussian profiles of the leptons.

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References

1. C.R. Schmidt, M.E. Peskin, Phys. Rev. Lett. **69**, 410 (1992)
2. E.D. Commins et al., Phys. Rev. A **50**, 2960 (1994)
3. J. Bailey et al., J. Phys. G **4**, 345 (1978)
4. Particle Data Group, D.E. Groom et al., Eur. Phys. J. C **15**, 1 (2000)
5. K.F. Smith et al., Phys. Lett. B **234**, 191, 2885 (1990); I.S. Altarev et al., Phys. Lett. B **276**, 242 (1992)
6. E.P. Sahabalin, Sov. J. Nucl. Phys. **28**, 75 (1978); J.F. Donoghue, Phys. Rev. D **18**, 1632 (1978); A. Czarnecki, B. Krause, Phys. Rev. Lett. **78**, 4339 (1997); I.B. Khriplovich, Yad. Fiz. **44**, 1019 (1986) (Sov. J. Nucl. Phys. **44**, 659 (1986)); Phys. Lett. B **173**, 193 (1986); A. Czarnecki, B. Krause, Acta Phys. Polon. B **28**, 829 (1997); Phys. Rev. Lett **78**, 4339 (1997); S, Weinberg, Phys. Rev. Lett. **58**, 657 (1976); N.G. Deshpande, E. Ma, Phys. Rev. D **16**, 1583 (1977); for a recent review, see H.Y. Cheng, Int. J. Mod. Phys. A **7**, 1059 (1992); G.C. Branco, M.N. Rebelo, Phys. Lett. B **160**, 117 (1985); J. Liu, L. Wolfenstein, Nucl. Phys. B **289**, 1 (1987); C.H. Albright, J. Smith, S.H.H. Tye, Phys. Rev. D **21**, 711 (1980); D. Atwood, S. Bar-Shalom, A. Soni, Phys. Rev. D **51**, 1034 (1995); E. Iltan, J. Phys. G **27**, 1723 (2001); Phys. Rev. D **65**, 073013 (2002); Y. Liao, X. Li, Phys. Rev. D **60**, 073004 (1999); D.G. Dumm, G.A. G Sprinberg, Eur. Phys. J. C **11**, 293 (1999); A. Soni, R.M. Xu, Phys. Rev. Lett. **69**, 33 (1992)
7. V. Baryshevsky, hep-ph/0402209; O. Lebedev, K.A. Olive, M. Pospelov, A. Ritz, Phys. Rev. D **70**, 016003 (2004); A.G. Rodriguez, M.A.H. Ruiz, A. Santiago, Phys. Rev. D **69**, 073008 (2004); A. Gutierrez-Rodriguez, M.A.Hernandez-Ruiz, L.N. Luis-Noriega, Mod. Phys. Lett. A **19**, 2227 (2004); R.E. Mischke, AIP Conf. Proc. **675**, 246 (2003); D.A. Demir, Oleg Lebedev, K.A. Olive, M. Pospelov, A. Ritz, Nucl. Phys. B **680**, 339 (2004); M. Endo, M. Kakizaki, M. Yamaguchi, Phys. Lett. B **583**, 186 (2004); D. Kawall, F. Bay, S. Bickman, Y. Jiang, D. DeMille, Phys. Rev. Lett. **92**, 133007 (2004); I. Masina, C. Savoy, Phys. Lett. B **579**, 99 (2004); J. Hisano, Y. Shimizu, Phys. Lett. B **581**, 224 (2004); A. Syntfeld, H. Mach, W. Kurcewicz, B. Fogelberg, W. Pociennik, E. Ruchowska, Phys. Rev. C **68**, 024304 (2003); T. Suzuki, H. Sagawa, K. Hagino, Phys. Rev. C **68**, 014317 (2003)
8. B. Dutta, R.N. Mohapatra, Phys. Rev. D **68**, 113008 (2003)
9. E. Iltan, Phys. Rev. D **64**, 013013 (2001)
10. E. Iltan, JHEP **065**, 0305 (2003)
11. E. Iltan, JHEP **0404**, 018 (2004)
12. N. Arkani-Hamed, M. Schmaltz, Phys. Rev. D **61**, 033005 (2000); N. Arkani-Hamed, Y. Grossman, M. Schmaltz, Phys. Rev. D **61**, 115004 (2000)
13. E.A. Mirabelli, M. Schmaltz, Phys. Rev. D **61**, 113011 (2000)
14. W.F. Chang, I.L. Ho, J.N. Ng, Phys. Rev. D **66**, 076004 (2002)
15. G.C. Branco, A. Gouvea, M.N. Rebelo, Phys. Lett. B **506**, 115 (2001)
16. W.F. Cang, J.N. Ng, JHEP **0212**, 077 (2002)
17. B. Lillie, J.L. Hewett, Phys. Rev. **68**, 116002 (2003)
18. Y. Grossman, G. Perez, Phys. Rev. D **67**, 015011 (2003); hep-ph/0303243
19. A. Delgado, A. Pomarol, M. Quiros, JHEP **0001**, 030 (2000); Y. Grossman, Int. J. Mod. Phys. A **15**, 2419 (2000); D.E. Kaplan, T.M. Tait, JHEP **0111**, 051 (2001); G. Barenboim et al., Phys. Rev. D **64**, 073005 (2001); W.F. Chang, I.L. Ho, J.N. Ng, Phys. Rev. D **66**, 076004 (2002); W. Skiba, D. Smith, Phys. Rev. D **65**, 095002 (2002); Y. Grossman, R. Harnik, G. Perez, M.D. Schwartz, Z. Surujon, hep-ph/0407260; P. Dey, G. Bhattacharya, Phys. Rev. D **70**, 116012 (2004); Y. Nagatani, G. Perez, hep-ph/0401070; R. Harnik, G. Perez, M. D. Schwartz, Y. Shirman, hep-ph/0411132
20. E. Iltan, H. Sundu, Acta Phys. Slov. **53**, 17 (2003)
21. M. Deile et al., Muon g-2 Collaboration, Nucl. Phys. Proc. Suppl. **116**, 215 (2003)